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SOCRATES IN BABYLON

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Introduction

Socrates never visited Babylon. He probably did not even know that in the Old Babylonian period about 1,000 years before his birth Babylonian scribes produced texts that about 2,000 years after his death would be known, using a seemingly genuine Greek term, as Babylonian mathematics. The focus of this paper then is not the question of where Socrates traveled during his lifetime but rather whether it is feasible to assume that a Babylonian scribe argued about mathematics in the same way that Socrates as a spokesman of Plato did 1,000 years later.

This question of what Babylonian scribes were able to establish is not in the first place a question about historical facts but rather an epistemological question. Is Greek mathematics a creation sui generis or can it be reasonably compared with the earlier Babylonian mathematical tradition? Is "Socrates in Babylon" a possible historical scenario.

Greek mathematical proofs

According to a widely held opinion ancient Greek mathematicians were the first to use proofs for assuring the truth of mathematical statements. Up to present times mathematical proofs in the Greek tradition serve as a normative model for representing deductive reasoning. There is an uninterrupted tradition reaching from the *Elements* of Euclid to modern mathematics to construct relations between propositions by chains of arguments arranged according to this model, and, as a matter of fact, neither Egyptian nor Babylonian mathematical texts contain anything comparable to a Greek proof.

As a consequence, until the first half of the 20th century the much older compilations of mathematical problems and problem solutions in Egypt and Babylonia were considered as mere collections of rules known from practical experiences. Hermann Hankel in his famous book on the history of ancient mathematics (Hankel 1874), for instance, divided this history into a prescientific and a scientific era beginning with Greek mathematics. In his monumental reference work on the history of mathematics Moritz Cantor in 1880 came to believe that in Egypt there must have existed a still undiscovered theoretical textbook on mathematics. But he remarked: "It is not so as though we are thinking of a theory in the modern sense. If one would not even simply consider evidence sufficient instead of any proof, proofs will mostly have been carried out inductively, possibly also on the basis of insufficient induction" (Cantor 1880, p. 63). Johan Ludvig Heiberg identified as late as 1925 ancient science with Greek science. He wrote about the Greeks: "Their contacts with the old civilization in Egypt and Mesopotamia may at best have challenged and provided them with material; but the Orient, bound by religion, was unable to make science of it; this only the free-thinking Ionia could accomplish." (Heiberg, 1925, p. 1) For all these authors Socrates in Babylon would have been an impossible scenario.

The challenge of Babylonian mathematics

This belief was substantially challenged in the 1920s when cuneiform tablets were identified that obviously held the solution to problems implicitly representing second degree equations. Since then hundreds of tablets with more or less complex mathematical problems have been translated and interpreted documenting what is now called Babylonian mathematics. In spite of the fact that the mathematical cuneiform tablets do not contain proofs as they occur in Euclid's *Elements*, most historians of mathematics working on Babylonian sources were soon convinced that they implicitly represent mathematical derivations comparable with the derivations in Greek mathematics and had even possibly influenced them.

This reinterpretation of the early history of mathematics initiated a vivid debate about questions such as whether Babylonian mathematics can be considered as real mathematics, whether it had any influence on Greek mathematics, whether it is justified to translate Babylonian and Greek mathematics into modern formalism, and so on (Høyrup 1996, pp. 11-17). Today, the debates have calmed down. While many historians of Greek mathematics still insist that proofs are the most important achievement in the history of mathematics, many historians of Babylonian mathematics simply assume implicitly by using modern mathematical formalisms that the solutions of the Babylonian problems are the result of essentially the same kind of derivations as those of the mathematical tradition going back to the Greeks.

Thus, essential problems remained widely unsolved. The Babylonian scribes who wrote the mathematical tablets do not provide us with deductive reasoning in any form. In nearly all cases they do not even formulate general rules how to solve the problems. They simply state the problem and inform us about the arithmetical operations to be performed with the given numerals to achieve the solution. But the complexity of some of the problems and their solutions make it impossible to believe that they should not be the result of sophisticated deductive reasoning. But what type of reasoning is it that might be hidden behind the succinct formulations of problems and problem solutions on the Babylonian mathematical tablets?

Fortunately, there are certain tablets which, if analyzed carefully, allow us a glimpse behind the curtain. These relatively few examples are worth studying meticulously as they allow some conclusions to be made about the reasoning that led to the solutions of the given problems. But before we examine two of these tablets let us take a closer look at the Greek tradition from a particular point of view.

Socrates and the slave

The Greeks were well aware of the special status of their mathematical propositions. They believed that in a certain sense they express something that is necessary as it is and cannot be other than it is. But they did not necessarily consider this status as depending on the form of representation that is transferred to us by Euclid. This is made evident, for instance, by the well-known dialog about the nature of mathematics in Plato's *Meno*.

In this dialog, Socrates maintains in a debate with his interlocutor Meno that mathematical knowledge is based on inherited ideas. He expresses Plato's opinion that we do not acquire these ideas but simply remember them. Socrates argues that an uneducated slave is in the same way capable of inferring mathematical knowledge as an educated Greek citizen. To convince Meno of the truth of his claim he calls a slave and demonstrates that even this uneducated man is able to find out that the square over the diagonal of a square has twice the area of the square itself.

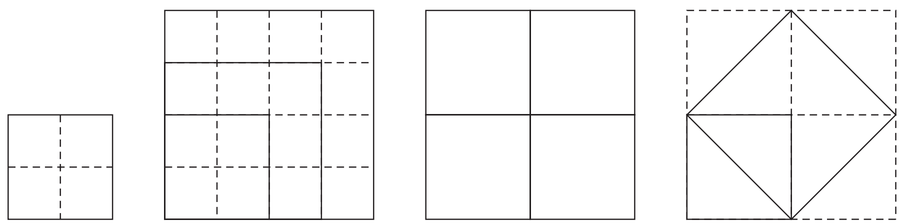


Figure 1: The drawings of Socrates

The dialog proceeds as follows (see figure 1). Socrates shows the slave the drawing of a square then divides it by halving each side into four equal smaller squares. He asks him to imagine that the length of each side of the original square is two feet, now divided into two parts of one foot length each. By asking several simple questions Socrates guides the slave to the insight that the area of the square must be four square feet.

Socrates asks now whether it would be possible to draw a square with twice the area, that is a square with an area of eight square feet, and how long the sides of such a square would have to be. He first receives the wrong answer that the sides must also be twice as long. Socrates constructs this larger square and guides him with further questions to the result that a square with sides twice as large does not have twice but rather four times the original area. Now the slave decides that a square with sides of three feet would have an area of eight square feet, but again guided by questions of Socrates he has to admit that this square would have nine square feet and thus again would not double the area of the square with sides of two feet.

The slave is now convinced that he does not know how long the side would have to be to give a square that has twice the area of a square with sides of two feet. Socrates again draws a square with sides to be imaged as two feet long, complementing it with three further squares of the same size to a square with sides of four feet. But now he also inserts a diagonal into each square so that these diagonals themselves form a square and guides the slave by questions concerning the number of triangles in the original square of two feet side length and the number of triangles in the square of diagonals to the insight that this latter square has the required property of doubling the area of the original square.

We do not need to deal here with the question of whether this dialog really confirms Plato's idea that gaining knowledge is closely related to a process of remembering inborn ideas. It is obvious that the essential steps of the solution to the problem do not come from the slave but from Socrates. He is the one who guides the slave by a sequence of simple questions in narrowly defined steps to count the partial areas of a square divided into smaller ones in order to find the correct area of squares with different sides. He also is the one who finally draws the diagonals forming the square that solves the given problem.

What is remarkable about Socrates' questioning of the slave and the interpretation of the scene in the dialog of Socrates and Menon is that they share the knowledge that allows them to decide whether a mathematical inference leads to a true result. They share the same reasoning, that is the reasoning of mathematical proofs. The dialog can be considered as a specific representation of a mathematical proof of the statement that the square of the length of the diagonal of a square is twice the square of the length of its sides, or rather, an Euclidean proof can be considered as a specific representation of deductive reasoning which does not depend on the specific form of how it is written down. This immediately raises the question of whether the first occurrences of proofs in the way they were written down by Euclid can really be identified with the origin of deductive reasoning and, in particular, of mathematics itself.

Babylonian calculation of the diagonal of a square

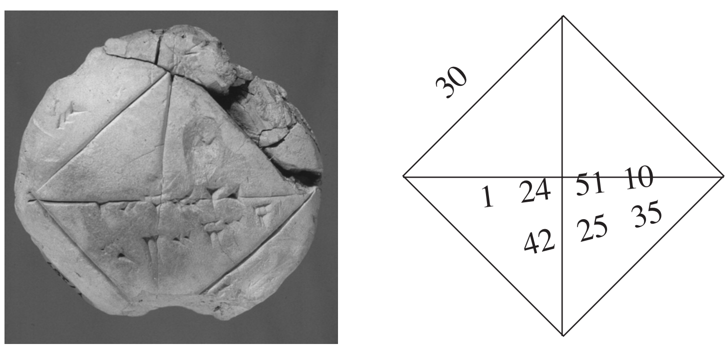


Figure 2: Old Babylonian practice tablet

Given this situation, the relatively few examples of Babylonian mathematics that allow for some conclusions about the reasoning that led to the solutions of the given problems are worth studying meticulously. One of these examples represented by two seemingly unrelated tablets is closely associated with the dialog of Socrates and the slave.

The first of these tablets (YBC 7289) is of unknown provenience hosted in the Yale Babylonian Collection and shows the typical round shape of a practice tablet of the disciple of an Old Babylonian scribal school, thus dating to the first half of the second millennium B.C. The tablet shows only the drawing of a square with its two diagonals inscribed. The length of the sides of the square is indicated by the numeral 30 written close to one of them. The numerals 1 24 51 10 are written along one of the diagonals. Underneath these numerals is written the sequence 42 25 35.

These sequences of numerals represent, of course, numerical notations in the celebrated sexagesimal positional system of numeration. Each numeral in a sequence represents 60 times the value of the next numeral of the sequence. The system does not allow the absolute value of the whole sequence to be indicated. The length 30 of the side can thus be interpreted as 30 of some length unit, but can also be read as 30 times 1/60, that is 1/2, or as 30 times 60, that is 1,800, or as 30 times the square of 60, that is 108,000 in the decimal system. Correspondingly, the sequence 42 25 35 can represent the number 42 plus 25 times 1/60 plus 35 times 1/3600, that is approximately 42.43 in the decimal system, but can also represent the number 42 times 3,600 plus 25 times 60 plus 35, that is 152,735 in the decimal system.

There is a simple relation between the two numerical notations written inside the square. Halving the number represented by the sexagesimal notation 1 24 51 10 written along the diagonal results in the sequence 42 25 35, that is the sexagesimal notation written underneath the first one. Furthermore, squaring the number represented by the sexagesimal notation 1 24 51 10 result in the sexagesimal notation 1 59 59 59 38 1 40 representing a number that is extremely close to the number 2 or to the number 2 multiplied by some power of 60. If we choose the absolute value of the sexagesimal notation 1 24 51 10 appropriately it thus turns out that it represents an extremely precise approximation of the square root of 2.

In fact, if we raise the last sexagesimal digit of the notation by one to 1 24 51 11, the sexagesimal notation of its square is 2 0 0 2 27 44 1. We will not discuss here the puzzling fact that seemingly this sexagesimal notation could not be written with the Old Babylonian cuneiform numerals since there was no sign for representing zero. This notation would have been written as 2 2 27 44 1 usually without any indication of the two zeros thus resulting in a totally ambiguous notation. But the comparison of the anachronistic notation 2 0 0 2 27 44 1 with the square 1 59 59 59 38 1 40 of the number given by the notation 1 24 51 10 written on the tablet is the closest approximation of the square root of 2 that could be written with 4 digits of the Old Babylonian sexagesimal positional system of numerical notation. In decimal notation it differs from the square root of 2 only by approximately 0.0000006.

What conclusions can be drawn from this school tablet concerning mathematical inferences that, as far as we know, have never been written down explicitly but may have been performed in the heads of the disciple or the teacher?

Since multiplying the length of the side of a square with the square root of 2 results in the length of the diagonal it is obvious that the drawing on the practice tablet and the numerical notations associated with the drawing served somehow to teach a disciple how to calculate the length of the diagonal of a square. The simplest interpretation of the tablet suggested by the drawing and the numerical notations is that the teacher instructed his disciple to calculate the diagonal of a square by multiplying the length of its side with 1 24 51 10. This interpretation is strongly supported by the occurrence of this numerical notation in one of the so-called coefficient lists (YBC 7243, see Robson 1999, p. 44). The coefficient is designated there as *ṣi-li-ip-tum* íb-si$_8$, that is, as the “diagonal of the square”. If we accept the common belief that such lists of coefficients were used for looking up factors by which certain figures have to be multiplied in order to solve specific problems, the practice tablet seems to represent nothing but the application of the rule that the side of a square has to be multiplied with 1 24 51 10 in order to achieve the length of its diagonal. Whether the disciple has to learn the coefficient by heart or whether he is allowed to look the factor up in a list of coefficients is unimportant given the situation that what the disciple essentially has to learn is to apply correctly a simple arithmetical rule. That the length of the side of the square given on the practice tablet makes the multiplication particularly simple supports the assumption that the purpose of the exercise documented by the practice tablet is not to train arithmetical operations but rather to learn how to solve the geometrical problem.

This interpretation leaves a crucial question open: Where did the teacher get the rule from? Has he also simply learned this rule? But who then was the first to establish the rule? Who calculated the numerical notation representing the square root of 2 and inserted it into the list of coefficients? Two common answers to such questions can easily be rejected.

The traditional answer based on the belief of the superiority and uniqueness of the achievements of Greek mathematics would be that the rule was found by experience. This answer, however, seems unacceptable given the enormous precision of the coefficient which is applied. Nobody could measure at that time the length of a diagonal with the precision given by the approximation of the square root of 2 as documented by the present practice tablet. For instance, if we assume that the given length of the side of the square were 30 cm, its diagonal would have had to be measured with a precision of one ten thousands of a millimeter to find a factor of the given precision.

A more recent answer to the question follows from the common belief that the scribes of the Old Babylonian period who wrote the so-called mathematical tablets knew the theorem of Pythagoras and that they derived the rule to multiply the length of the side with the square root of 2 in order to achieve the length of the diagonal from this theorem. But even if the scribes knew this theorem in some form, what evidence do we have that they derived rules for the solution of problems from theorems in a way one must have in mind to believe that the use of the the square root of two on the practice tablet was based on the knowledge of the theorem of Pythagoras. The scribes neither wrote down derivations of rules nor even explicitly formulated theorems and rules. They must have derived everything in their head that the Greeks had written down and complemented with drawings in order to manage complex derivations from their theorems. But why should we assume such superior competence of Babylonian scribes living 1,500 years before the Greek mathematicians, which could have enabled them to derive the alleged knowledge about solution procedures from geometrical theorems such as the theorem of Pythagoras, doing this all by heart?

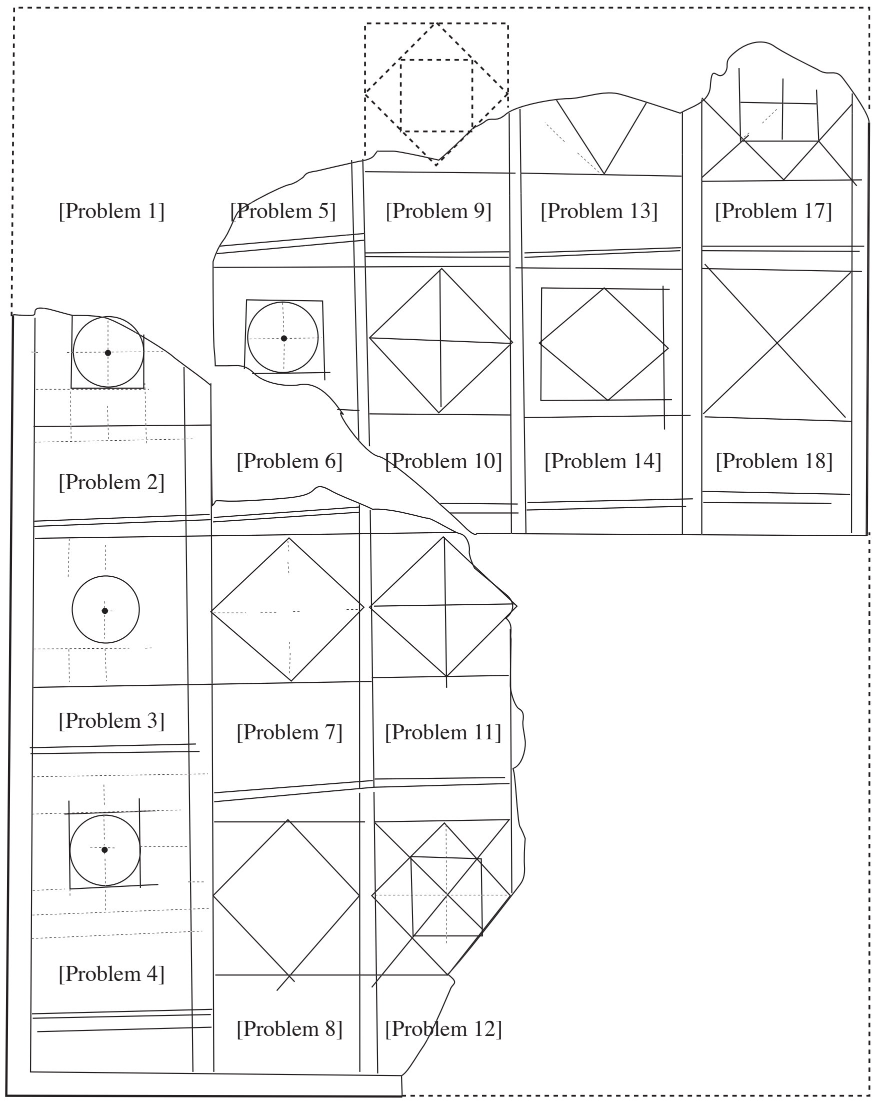


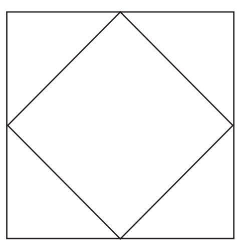
Figure 3: Scheme of the obverse of a teacher's didactic manual

A Babylonian teacher's manual

Fortunately, in the British Museum two substantial fragments of another tablet exist (BM 15285) which may shed some light on the context of teaching and learning of the present practice tablet. Again, this tablet can be dated to the Old Babylonian period in the first half of the second millennium B.C. and also comes from a school context. The first fragment was published in Neugebauer's pioneering edition of mathematical cuneiform tablets (Neugebauer 1935/37, vol. 1, pp. 137-142 and vol. 2, plates 3 and 4), the second fragment was later discovered in the archives of the British Museum and published by Saggs (Saggs 1960). The complete tablet probably contained 41 drawings of nested figures inscribed into a square with a side of length 1, each of them complemented by a short description of the construction of the drawing ending always with the same question: What are the areas?

The format of the tablet, its careful preparation, and the arrangement of the problems make it unlikely that the tablet should be the exercise of a disciple. It rather seems to represent something like a didactic manual of the teacher. The problems do not follow each other in an arbitrary order. The order of drawings indicates an intimate connection between consecutive problems although the meaning of this order is not always obvious. There is, however, a sequence of six problems, starting with the seventh and ending with the twelfth problem of the obverse of the tablet, that can be interpreted as steps initiating a specific learning process in the mind of the disciple. The drawings alone indicate the connection exhibiting essentially the same basic scheme with an increasing complexity of additional lines drawn into a square with a side of 1 UŠ length.

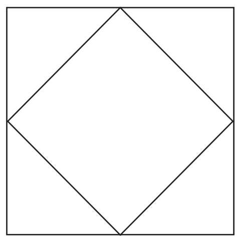
The basic scheme of these drawings, squares inscribed into the basic outer square with different sizes and partly turned 45 degrees, is obviously related to the practice tablet discussed above. But the drawings of the sequence also show a great similarity to the drawings Socrates used in Plato's dialog with Meno to make the slave understand that the square of the diagonal of a square has twice the area of the original square.



1UŠ, (the side of) a square.
In the middle I drew another square.
The square that I drew
touches the outer square.
Their area what?

Figure 4: Drawing and problem 7 of the teacher's didactic manual

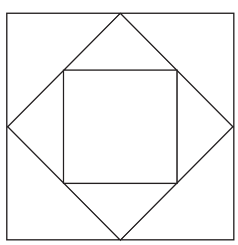
The sequence of problems starts with a single square inscribed into the basic outer square. The corners of the inscribed square touch the sides of the outer square in the middle so that the outer square becomes the square of the diagonal of the inner square, but the diagonals are not drawn. The accompanying text begins with the description of this construction. The following question asking for the areas can be considered as the central question of the whole sequence of problems. A correct answer would be that the inner square has half the area of the outer square and that the four triangles at the edges resulting from the construction of the inner square together cover the other half of this area. This answer would obviously solve the problem with which Socrates confronted the slave, and at the same time explain how the Babylonian scribes could find the rule to multiply the side of a square with the square root of 2 in order to find the length of the diagonal. However, the text of the Babylonian teacher seems not to assume that the disciples or, at least, some of them would be immediately able to find the answer by merely looking at the drawing. Thus, the following problems provide some help for answering the central question concerning the relation of the area of a square and the area of the square of its diagonal.



1UŠ, (the side of) a square.
In the middle 4 triangles,
1 square. The square that I drew
touches the other square.
Their area what?

Figure 5: Drawing and problem 8 of the teacher's didactic manual

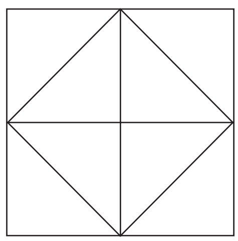
The second problem concerns the same figure, but the description of its construction is different from the description in the first problem. Now the four triangles at the corners are drawn and the inner square results from the construction of these triangles. This variation of the description of how the same figure can be constructed in a different way can be considered as a didactic means to provoke a reinterpretation of the visual cues of the drawing in the mind of the disciple. In fact, while the inner square drawn without the diagonals provides no obvious hint in helping to find its area, the triangles at the corner directly suggest how their area can be found by completing them to squares



1UŠ, (the side of) a square.
In the middle I drew a square.
The square that I drew
touches the square.
In the middle of the second square
I drew a third square. That I drew
touches the square.
Their area what?

Figure 6: Drawing and problem 9 of the teacher's didactic manual

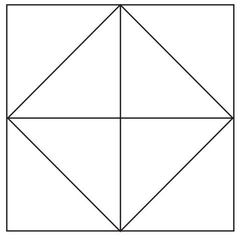
The following problem provides a further hint. Now a further square is inscribed into the inner square so that the side of the previously inserted square becomes its diagonal, thus repeating the original construction of an inner square touching the outer one. The description of the construction follows precisely this scheme. The text returns to the description of the first construction, but now literally repeating it for the innermost square. What did the teacher expect from this apparent increase of the complexity of the figure? The answer to this question is obvious if we remember the error of the slave in his dialog with Socrates that doubling the side of a square would also double its area. This seemingly self-evident assumption may be an obstacle in accepting that the inscribed squares should cover half of the area of the outer square since the side of the inscribed square is evidently longer than half of the side of the outer square. The repetition of the operation of inscribing a square results in a square with half the side of the outer square, but its area covers obviously less than one half of the area of the original outer square. Again, the disciple learns to view the original problem from a different perspective, that is, to consider the relation between the lengths of the sides of different squares and the corresponding areas in order to get a better understanding of the relation between lengths and sides.



1UŠ, (the side of) a square.
In the middle I drew 8 triangles.
Their area what?

Figure 7: Drawing and problem 10 of the teacher's didactic manual

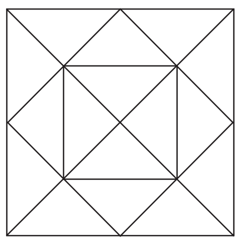
What should the teacher do if, in spite of the deepened understanding of the original problem a disciple may have gained, he still does not see how to find the area of the inscribed square? The following two problems provide a systematic answer. The area has to be divided into smaller areas the size of which can more easily be determined. This division into areas that could easily be calculated was the main technique used for hundreds of years by the surveyors in Mesopotamia to determine the areas of agricultural fields. The description of the construction says that the original outer triangle is divided into eight triangles, implicitly assuming that the eight triangles are equal. The resulting figure contains the figure of the original problem of the whole sequence, but it is left to the disciple to find that out, again by viewing the figure from another perspective.



1UŠ, (the side of) a square.
In the middle I drew a square.
The square that I drew
touches the square. In the middle
of the square
I drew 4 triangles.

Figure 8: Drawing and problem 11 of the teacher's didactic manual

The next problem provides precisely this information as a further hint. The drawn figure is the same, but the description of the construction returns now to the original description of the inscribed square, now adding to it that four triangles are inscribed into the inner square. This solves the initial problem completely. Mere counting of the triangles in the original square and the inscribed square shows that the inscribed square covers half of its area.



1UŠ, (the side of) a square.
Inside [I drew] 16 triangles.
Their area what?

Figure 9: Drawing and problem 12 of the teacher's didactic manual

A final problem ends the sequence. Returning to the repeated operation inscribing the square twice so that the second square has half the length of the side of the original outer square, the figure is divided into 16 equal triangles. Eight of them cover the first inscribed square and four of them the second inscribed square. The rule is thus made obvious that with every inscribed square the area is decreased to one half of the previous one.

Geometrical intuition in Greece and in Babylonia

What does the teacher who left us a didactic manual – let us call him the Babylonian Socrates – tell us about the nature of mathematics? His manual surely warns us to be careful with answers to the questions of when mathematics was created and what kind mathematical competence existed before mathematics was created. Obviously, the method by which the Greek Socrates helped the slave gain an understanding of how to double the area of a square was well known and already applied in the scribal schools of the Old Babylonian period, that is, more than one thousand years before the Greek Socrates was born. Not only the outcome that resulted from the questions in the manual of the Babylonian Socrates was the same as that resulting from Plato's dialog. They also shared a number of specific peculiarities which tell us something about a common background and specific differences of the Babylonian and the Greek way of dealing with geometrical problems.

Both the Babylonian and the Greek Socrates do not make their disciples derive the solutions to their answers from mathematical knowledge in the form of propositions. They rather refer to the geometrical content of their questions by showing geometrical figures. Both refer to the figures by describing how they are constructed, a procedure in the Greek mathematical tradition that was later reflected in the prominent role of the construction of figures in Euclid's system of proofs. Both the Babylonian and the Greek Socrates try then to get their disciples to look at the figures from different perspectives, to look at them as if they were constructed in a different way, essentially starting with those parts that make up the answer to the questions, that is, the square of the diagonal. They expect that, at least after they had added further lines to the figure, they may get the disciples to find the right answer by comparing the adequate parts of the figures.

But does this mean, that Babylonian and Greek geometry were built on the same ground? Surely not. The Greek Socrates on the one hand knew the theorem of Pythagoras and how to derive from it the construction of a square with twice the area of a given square. He disguises this knowledge behind a strategy of asking questions which enabled the uneducated slave to see that his guessed solutions were wrong. He progressively enriched his figure until it finally contained the solution, that is the square of the diagonal, so that the slave with his limited mathematical knowledge had only to verify its correctness.

This tells us almost nothing about how Socrates himself would have arrived at the solution without his knowledge of the theorem of Pythagoras, nor about the mathematical knowledge the slave would have needed to find the solution without the guiding questions of Socrates. His dialog with the slave was an artificial construction that should lead the slave to accept the correctness of the solution to a specific problem, pretending that the slave himself was finding the answer.

The Babylonian Socrates, on the other hand, when he compiled his manual surely did not intend to write down how to solve the problem of finding the length of the diagonal of a square as is documented by the practice tablet discussed above (YBC 7289). He probably had only vague ideas about what he had to teach his disciples to enable them to solve such problems. Our discussion of part of his manual shows that it documents general didactic means to enable the disciples to derive mathematical insights and rules for solving problems from arguments about geometrical constructions.

The tablet with the manual remains a unique finding among the great variety of some hundred mathematical cuneiform tablets excavated so far. This uniqueness probably results from the fact that the teacher wrote down what usually was based on an oral tradition of teaching practices. The sequence of questions did not serve as a means of teaching how to solve a specific problem but rather to develop the geometrical intuition of his disciples. It may be fortuitous that the sequence of six problems of the manual discussed here is so closely related to the calculation of the diagonal of a square on a practice tablet. Other problems of the manual do also contribute to learning how to calculate this diagonal. There is, for instance, further down on the tablet a square divided into 16 equal subsquares and the trivial problem is posed to calculate the areas. This problem would also fit nicely into the dialog of Socrates and the slave, but on the Babylonian tablet it is placed too close to the end (4th problem of the reverse) to be considered as being concerned with the sequence related to the diagonal of a square (7th to 12th problem of the obverse).

A comparison with the proofs in the *Elements* of Euclid, which became the model of deductive mathematical reasoning, also shows remarkable differences. It is true that, similar to Euclid's *Elements* where all proofs start with a construction of the figure that is subject of the proposition to be proven, the problems of the Babylonian manual also start with descriptions of the constructions of the figures. However, whereas the descriptions of constructions in Euclid's *Elements* tend to be unambiguous, which was made possible by the Greek invention of lettering the figures (Netz 1999), the descriptions in the Babylonian manual are elliptic and often understandable only together with the figures already drawn. Whereas the proofs in Euclid's *Elements* tend to use a stereotyped language for all technical terms, supported by the Greek invention of defining concepts, the Babylonian manual varies remarkably in the descriptions of the constructions by using grammatically inflected Akkadian terms or Sumerograms. The difference between Greek and Babylonian geometry, insofar as the first is represented by Euclid's compilation of Greek proofs and the second by the ingenious way of developing geometrical intuition for solving problems as is documented by the teacher's manual, can thus be summarized as being that of a linearized and canonized technique of representing geometrical intuition in written form opposed to an oral tradition of teaching geometrical intuition. This result supports recent attempts to reinterpret the so-called Babylonian algebra as actually not being based on algebraic methods but on a highly developed geometrical intuition (Høyrup 1990 and 2002).

Let us finally return briefly to the question of what Socrates in Babylon tells us about the nature of early mathematics. Given that Socrates and the Babylonian scribe both based their arguments on geometrical intuition and that the deductive form that Greek mathematics displays in Euclid's *Elements* can be conceived of as a canonized written representation of inferences based on geometrical intuition, Greek mathematics can no longer be considered as a creation *sui generis*. Nevertheless, geometrical intuition is no universal human resource. Modern studies of ethnomathematical capabilities have made evident that a rich variety of different geometrical capabilities can be developed under different cultural conditions (Gerdes 1990; Ascher 1994; D'Ambrosio 2006).

This variety is even greater if we look at the historical development of geometry and geometrical techniques. Thus, Greek geometry was dependent on an intuition developed on the basis of handling a highly developed technique of constructing figures, including an understanding of the differentiation of geometrical shapes by angles, congruence, and similarity.

By contrast, Babylonian geometry was based on a tradition of one thousand years of surveying fields. The surveyors did not care for angles and similarity. Their technique of surveying was based on the division of irregularly shaped fields into triangles and quadrangles, and on a procedure of calculating the partial areas that depended only on length measurements. This made any concept of angle, congruence, and similarity irrelevant to them. The area of a triangular field was calculated as half the product of its two shorter sides, the area of a quadrangle as the product of the means of opposite sides. From the viewpoint of Euclidean geometry these arithmetical procedures appear as mere approximations of the "true" Euclidean area, but in the context of the Babylonian tradition the concept of area was based on them resulting in some kind of "non-Euclidean" geometry which combines the additivity of the area concept with the surveyors' concept of the quantity of an area (Damerow 2001).

Babylonian mathematics adopted not only the technical terminology of the surveying practitioners, but also their area concept which was independent of any concept of an angle (Gandz 1929). The consequences were to a great extent identical with those of Euclidean geometry. The teacher's manual discussed here can be seen as a kind of missing link between administrative documents of the surveyors and the Old Babylonian mathematical texts. The manual shows, on the one hand, how the additivity of the area concept was used to develop a geometrical intuition beyond what was needed by the practitioners. It shows, on the other hand, how the additivity of the concept of area, applied to a variety of complex figures, can lead to solutions of the complex problems of Babylonian mathematics. Like the majority of those more complex problems that occur on the mathematical cuneiform texts, none of the problems of this manual have anything to do with the concept of an angle so that the non-Euclidean geometrical intuition of the Babylonian Socrates remains hidden. But if we could listen to the unwritten dialogs of the Babylonian Socrates and his disciples we would also hit on such strange challenging exercises as the application of the theorem of Pythagoras to triangles other than right-angled ones (YBC 8633, see Damerow 2001, pp. 244f.) or the task of calculating the length of the dividing line of a quadrangle which, under an arbitrary angle, is cut into two equal parts (YBC 4675, see Damerow 2001, pp. 280-286). Both problems are sheer nonsense in the framework of Euclidean geometry but fit perfectly into the geometrical intuition and knowledge of the Babylonian Socrates – but this is another story to be told.

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